Sections 2.1 & 2.2 - I.C.E. **Name:** _____

SHOW ALL WORK!!!! ☺

Assume you cannot use a graphing calculator for these problems.
The ones on which you CAN use a graphing calculator are starred.

For questions 1 and 2, determine the left hand and right hand behavior of the graph of function. Fill in the blank with the appropriate sign (positive or negative)

1. $y = -4x^3 - 2x^2 + 3$

as $x \to \infty$, $y \to$ __−__ ∞

as $x \to -\infty$, $y \to$ __+__ ∞

2. $3x^6 - 2x^5 + 4x^3 - x$

as $x \to \infty$, $y \to$ __+__ ∞

as $x \to -\infty$, $y \to$ __+__ ∞

3. $f(x) = -(-x^2 - 2x + 4)$ (for vertex, use $h = \frac{-b}{2a}$ and $k = f(h)$)

a) Vertex: $x^2 + 2x - 4$

$-\frac{b}{2a} = \frac{-2}{2} = -1$ $\quad f(-1) = 1 - 2 - 4 = -5$

$(-1, -5)$

b) x-intercepts:

$0 = x^2 + 2x - 4$

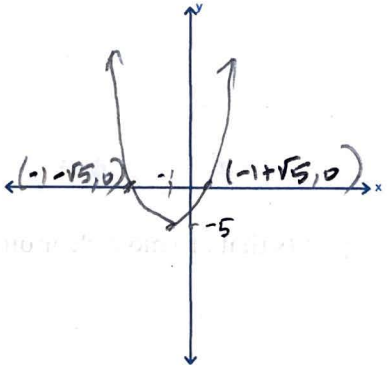
not factorable, so quadratic form:

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-4)}}{2(1)}$$

$$= -1 \pm \frac{2\sqrt{5}}{2}$$

$$= -1 \pm \sqrt{5}$$

c)



4. $f(x) = -x^2 + 5x + 24$ (find the vertex by completing the square)

a) Vertex:

$f(x) = -(x^2 - 5x - 24) = -\left(x^2 - 5x + \frac{25}{4}\right) - 25 - \frac{25}{4}$

$= -\left(x - \frac{5}{2}\right)^2 - \frac{125}{4}$

Vertex: $\left(\frac{5}{2}, \frac{125}{4}\right)$

b) x-intercepts:

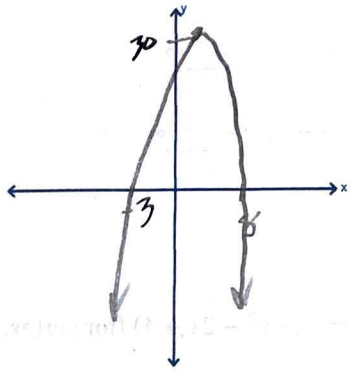
$0 = -(x^2 - 5x - 24)$

$0 = -(x - 8)(x + 3)$

$x = 8, -3$

$(8, 0), (-3, 0)$

c)



For questions 5 and 6, give the equations, in vertex form, for the following graphs.

5. Vertex: $\left(\frac{6}{5}, -\frac{2}{3}\right)$, passing through $(2, -4)$

$y = a(x - h)^2 + k$

$y = a\left(x - \frac{6}{5}\right)^2 - \frac{2}{3}$

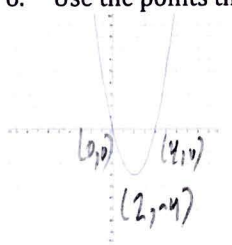
$-4 = a\left(2 - \frac{6}{5}\right)^2 - \frac{2}{3}$

$-4 = \frac{16}{25}a - \frac{2}{3}$

$-\frac{10}{3} = \frac{16}{25}a \Rightarrow a = -\frac{125}{24}$

$\therefore y = -\frac{125}{24}\left(x - \frac{6}{5}\right)^2 - \frac{2}{3}$

6. Use the points that are most clear on this graph:



$y = a(x - h)^2 + k$

$0 = a(0 - 2)^2 - 4$

$4 = 4a$

$1 = a$

$y = (x - 2)^2 - 4$

7. $f(x) = -x^3 + 8x^2 - 16x$

a) find the zeros and determine the multiplicity of each zero

$f(x) = -x(x^2 - 8x + 16) = -x(x-4)^2$

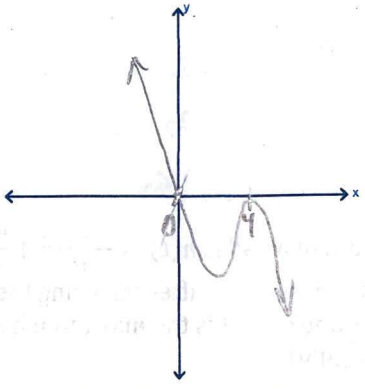
$x = 0$ mult = 1 $x = 4$ mult = 2

b) determine the left/right hand behavior for the polynomial

as $x \to -\infty$, $y \to +\infty$

as $x \to \infty$, $y \to -\infty$

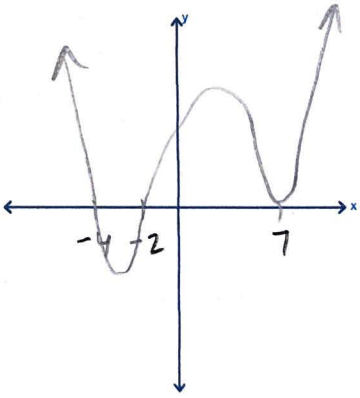
c) use this information to sketch a graph.



8. Find a polynomial of degree n that has the given zeros and then draw a sketch of your polynomial. You may leave it in factored form: you do not need to FOIL it out. (Many correct answers)

Zeros: x = -4, -2, 7 Degree: n = 4

$(x+4)(x+2)(x-7)^2$



9. Find a quadratic function **in standard form** whose graph has the given x – intercepts. (Many answers)

x-intercepts: (-3, 0) and (2, 0)

$f(x)=(x+3)(x-2)=x^2+x-6$ ← Standard form

factored form ↑

10. Write a quartic polynomial, $P(x)$, **in standard form** with the following conditions:

Zero at $x = 2$ (multiplicity 3),
Zeros at $\sqrt{5}$ (multiplicity 1) and $-\sqrt{5}$ (multiplicity 1)
y-intercept at (0,80)

$y=a(x-2)^3(x-\sqrt{5})(x+\sqrt{5})$
$y=a(x^2-4x+4)(x-2)(x^2-5)$
$y=a(x^3-4x^2+4x-2x^2+8x-8)(x^2-5)$
$y=a(x^3-6x^2+12x-8)(x^2-5)$
$y=a(x^5-6x^4+12x^3-8x^2-5x^3+30x^2-60x+40)$

$y=a(x^5-6x^4+7x^3+22x^2-60x+40)$

solve for a

$80=a(0+40) \Rightarrow a=2$

$y=2x^5-12x^4+14x^3+44x^2-120x+80$

*11. The path of a ball is $h(t)=-\frac{49}{10}t^2+\frac{98}{5}t+\frac{294}{5}$ where h is the height (in feet) and t is the number of seconds after throwing the ball. When (in seconds after release) does the ball hit the ground? What is the maximum height of the ball? (Do the work algebraically, then confirm graphically).

max: $h=\frac{-b}{2a}=\frac{-\frac{98}{5}}{2(-\frac{49}{10})}=-\frac{98}{5}\cdot-\frac{10}{98}=2$ secs

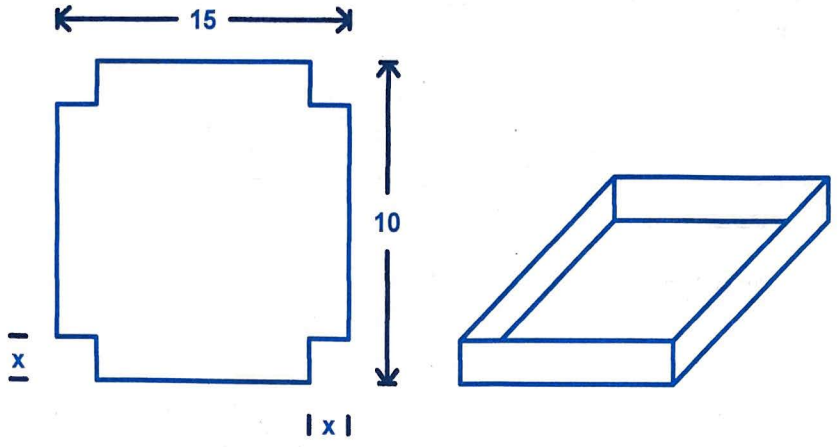
$k=f(2)=-\frac{49}{10}(4)+\frac{98}{5}(2)+\frac{294}{5}=\frac{392}{5}=78.4$ ft.

confirmed on calculator

Zeros: found on calculator

hits the ground after 6 seconds

*12. An open box with locking tabs is to be made from a square piece of material 15 inches on one side and 10 inches on the other, and this is to be done by cutting equal squares with side length x from the corners and folding up the sides.



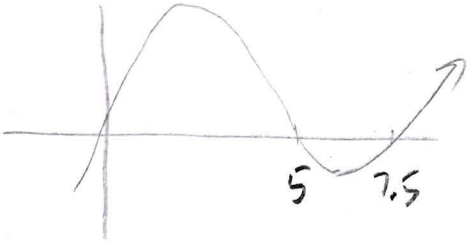
a) What is the volume of the box in terms of x? (you can leave in factored form)

$V(x) = (15-2x)(10-2x)x$

b) What is the domain of the function V?
(in other words, what values of x make this a box that can actually be built?)

$x > 0$ $\quad 15-2x > 0$ $\quad x < \frac{15}{2}$ $\quad 10-2x > 0$ $\quad x < 5$ $\quad$ all 3 must be true, so $0 < x < 5$

c) Sketch a graph of the function and find the value of x that will give the maximum volume:
(be sure to adjust your window so you can see the WHOLE graph!)



d) Use your calculator to find the following information:

Max volume will happen at x = 1.962 in $\quad$ max is $132.04\ in^2$

Maximum volume is _____

*13. Miles throws a softball; the table below shows the height $h(t)$ of the ball t seconds after it is thrown. Give the quadratic regression equation that best fits the data. Round the coefficients to three decimals.

t	0.25	0.5	1
$h(t)$	7	9	6

Quadratic model: $h(t) = -18.667t^2 + 22t + 2.667$